ISHair: Importance Sampling for Hair Scattering

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We present an importance sampling method for the bidirectional scattering distribution function (*bsdf*) of hair, based on the multi-lobe hair scattering model presented in [Sadeghi et al. 2010]. Our algorithm is efficient, easy to implement and it has no significant memory overhead or need for precomputation. We have integrated our method into both a research raytracer and a micropolygon based production renderer. Figure 1 compares the rendering quality of our method to statified uniform sampling for both direct (environment) lighting rendered with our production renderer and indirect lighting rendered with path tracing. In both cases, our method delivers significantly better image quality than uniform sampling using the same number of samples.

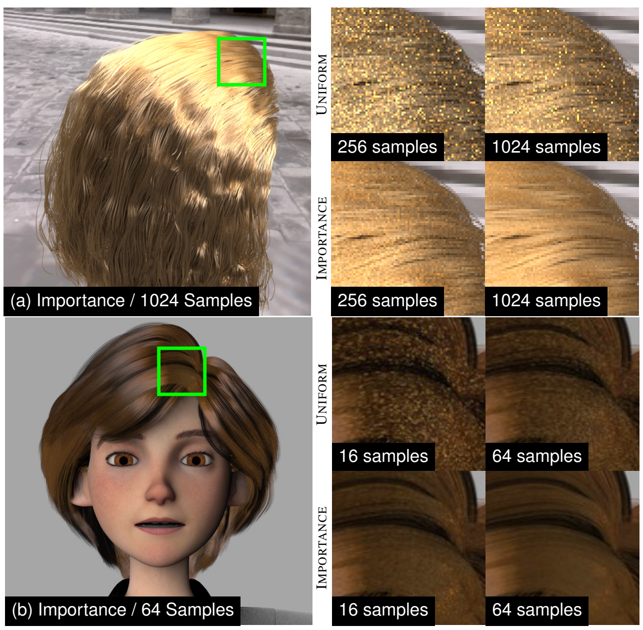

Figure 1: *Comparison of uniform sampling and our importance sampling method. (a). Global Illumination. (b). Direct Lighting.*

Hair Importance Sampling

Sadeghi et al. [2010] proposed an artist-friendly hair shading model, where they decomposed the hair scattering function $S(\theta_i, \phi_i, \theta_r, \phi_r)$ into four components: reflection (R), refractive transmission (TT), secondary reflection without glint (TRT-g) and *Glint* (g); each lobe was further factored as a product of a longitudinal term M and an azimuthal term N:

$$\begin{aligned} S(\theta_i, \phi_i, \theta_r, \phi_r) = \; & I_{\mathrm{R}} M_{\mathrm{R}} N_{\mathrm{R}} / \cos^2\theta_d + I_{\mathrm{TT}} M_{\mathrm{TT}} N_{\mathrm{TT}} / \cos^2\theta_d \\ & + I_{\mathrm{TRT}} M_{\mathrm{TRT}} N_{\text{TRT-g}} / \cos^2\theta_d + I_{\mathrm{TRT}} M_{\mathrm{TRT}} I_{\mathrm{g}} N_{\mathrm{g}} / \cos^2\theta_d \end{aligned}$$

For importance sampling, we want to sample the incoming direction ω_i with $p(\omega_i) \propto S(\theta_i, \phi_i, \theta_r, \phi_r)$ where θ_i and ϕ_i are the longitudinal and azimuthal components of ω_i. Since the longitudinal and azimuthal terms are independent, we can sample θ_i and ϕ_i separately then convert them into ω_i. The *pdf* of each sample is $p(\omega_i) = p(\theta_i)p(\phi_i)$. Importance sampling the hair scattering model is challenging because all the longitudinal terms are Gaussian functions of the longitudinal half angle and the azimuthal transmission and *Glint* terms are Gaussian functions of the relative azimuthal angle. Because Gaussians do not have a closed form anti-derivative, direct application of the inverse *cdf* sampling technique is nontrivial. To overcome this challenge, we approximate the Gaussian using Cauchy distribution

$$f(\gamma, x - x_0) = \frac{1}{\pi}\left[\frac{\gamma}{(x - x_0)^2 + \gamma^2}\right]$$

, a bell shaped function with a closed form anti-derivative: $P(x) = \frac{1}{\pi}\tan^{-1}\left(\frac{x - x_0}{\gamma}\right)$.

Sampling the individual lobes For the longitudinal terms defined as

$$M_{\mathrm{x}} = g(\beta_{\mathrm{x}}^2, \alpha_{\mathrm{x}}, \theta_h), \quad \mathrm{x} = \{\mathrm{R}, \mathrm{TT}, \mathrm{TRT}\}$$

where β_{x} and α_{x} are the variance and mean of the Gaussian; using the Cauchy approximation to the Gaussian, we are able to derive a closed form solution to importance sample θ_i as:

$$\theta_i = 2\beta_{\mathrm{x}} \tan(\xi(A - B) + B) + 2\alpha_{\mathrm{x}} - \theta_r$$

where ξ is uniformly drawn from range $[0, 1)$, $A = \tan^{-1}\left(\frac{\pi/4 + \theta_r/2 - \alpha_{\mathrm{x}}}{\beta_{\mathrm{x}}}\right)$ and $B = \tan^{-1}\left(\frac{-\pi/4 + \theta_r/2 - \alpha_{\mathrm{x}}}{\beta_{\mathrm{x}}}\right)$; with the *pdf*: $p(\theta_i) = \frac{1}{2\cos\theta_i(A - B)}\frac{\beta_{\mathrm{x}}}{(\theta_h - \alpha_{\mathrm{x}})^2 + \beta_{\mathrm{x}}^2}$. For the azimuthal terms, we apply a similar approach to derive closed form solutions to importance sample N_{TT} and N_g, both defined as Gaussian functions of the relative azimuthal angle ϕ. The remaining terms N_{R} and N_{TRT-g}, are straightforward to importance sample as they are approximated using $\cos(\phi/2)$. The complete derivation and solution to importance sample all the independent terms of the hair *bsdf* are provided in the supplemental material, along with the pseudocode of our algorithm.

Energy-based lobe selection is used to distribute samples amongst the different lobes. For each sample, we select a lobe with a probability proportional to its energy by building a *cdf* using the following lobe energy estimations:

$$\mathrm{E}_{\mathrm{R}} = 4\sqrt{2\pi}\beta_{\mathrm{R}} I_{\mathrm{R}} \qquad \mathrm{E}_{\mathrm{TT}} = 2\pi\beta_{\mathrm{TT}}\gamma_{\mathrm{TT}} I_{\mathrm{TT}}$$

$$\mathrm{E}_{\text{TRT-g}} = 4\sqrt{2\pi}\beta_{\mathrm{TRT}} I_{\mathrm{TRT}} \qquad \mathrm{E}_{\mathrm{g}} = 4\pi\beta_{TRT}\gamma_{\mathrm{g}} I_{\mathrm{TRT}} I_{\mathrm{g}}$$

Results

Figure 2 demonstrates our importance sampling algorithm is quite effective at distributing more samples towards regions with higher energy. This is also verified by rendering comparison between environment lighting, area lighting and bounce lighting of hair *bsdf* using stratified uniform sampling versus our importance sampling algorithm. In most of the directing lighting scenarios, our importance sampling algorithm yields better quality than uniform sampling with 4x number of samples. The improvement is even more significant in renders with multiple scattering and indirect lighting.

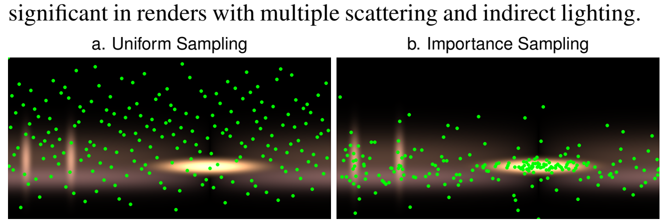

Figure 2: *Comparison of samples distributed using (a) uniform and (b) importance sampling.*

References

SADEGHI, I., PRITCHETT, H., JENSEN, H. W., AND TAMSTORF, R. 2010. An artist friendly hair shading system. *ACM Transactions on Graphics 29*, 4 (July), 56:1–56:10.

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